Note on up and down conversions in LLRF systems

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1 Introduction

The simplified LLRF system considered in this paper focuses on the up and down conversion happening in the signal chain. It consists of a receiver/transmitter, performing the transition from RF signals to intermediate frequency (IF) signals, and a controller, performing the transition from IF to base band and back to IF. The diagram below illustrates the succession of down conversion and up conversions.

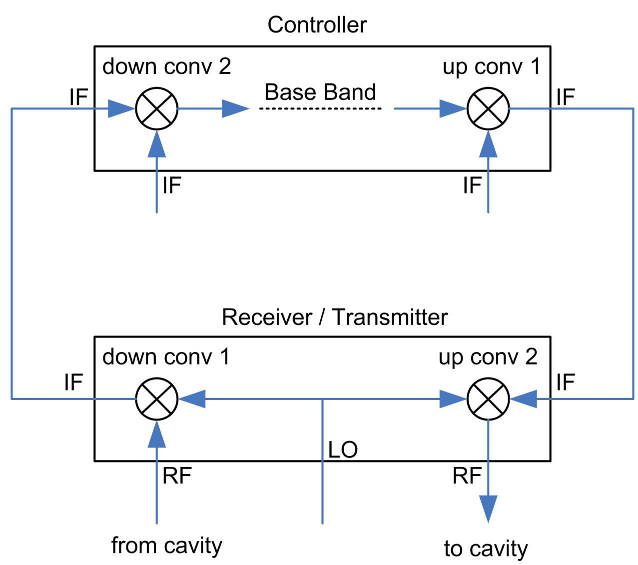

Figure 1: Up and down conversions in the LLRF signal path

In the case of HINS, RF=325 MHz, LO=338 MHz and IF=13 MHz. An RF signal of amplitude A_{RF}, frequency ω_{RF} and phase ϕ_{RF} is denoted as $A_{\mathrm{RF}} \sin(\omega_{\mathrm{RF}} t + \phi_{\mathrm{RF}})$. Unless specified otherwise, most examples will only focus on the frequency of the signal so the phase and amplitude information is discarded, and the following notation is used: $\sin \mathrm{RF}$ for simplicity. Similarly, $\sin \mathrm{LO}$ and $\sin \mathrm{IF}$ refer to signals at the local and intermediate frequency.

2 Trigonometric identities

The following trigonometric identities are used throughout this paper.

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \tag{1}$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b \tag{2}$$
$$\sin(a+b) = \sin a \cos b + \sin b \cos a \tag{3}$$
$$\sin(a-b) = \sin a \cos b - \sin b \cos a \tag{4}$$

and the converse identities

$$\cos a \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b) \tag{5}$$
$$\sin a \sin b = \frac{1}{2}\cos(a-b) - \frac{1}{2}\cos(a+b) \tag{6}$$
$$\sin a \cos b = \frac{1}{2}\sin(a+b) + \frac{1}{2}\sin(a-b) \tag{7}$$
$$\cos a \sin b = \frac{1}{2}\sin(a+b) - \frac{1}{2}\sin(a-b) \tag{8}$$

In the following four sections, the mathematical operation corresponding to the up and down conversions of the receiver and the controller will be derived.

3 Receiver down conversion

This is the down conversion 1 illustrated in figure 1. The operation consist of down converting the cavity RF signal to an IF and is typically implemented using an RF mixer followed by a band pass filter to discard the unwanted side band. The mathematical model consists of multiplying the RF signal by $\sin \mathrm{LO}$, and only keeping the term at the IF. To better illustrate the process, we consider an RF signal with a frequency offset Δf so that the RF signal is represented as $\sin(\mathrm{RF} + \Delta f)$, described as case 1. For completeness, the expressions will also be given for $\cos(\mathrm{RF} + \Delta f)$ (case 2).

case 1: $\sin(\mathrm{RF} + \Delta f)$
The down converted signal becomes:

$$\begin{aligned}
\sin(\mathrm{RF} + \Delta f) \times \sin \mathrm{LO} &= \frac{1}{2}\cos(\mathrm{RF} + \Delta f - \mathrm{LO}) - \frac{1}{2}\cos(\mathrm{RF} + \Delta f + \mathrm{LO}) \\
&= \cos(\text{RF-LO} + \Delta f) \\
&= \cos(\text{-IF} + \Delta f) \\
&= \cos(\mathrm{IF} - \Delta f)
\end{aligned}$$

Note that in the second line, the term in RF+LO is filtered out and the 1/2 amplitude coefficient is dropped out for lighter notations. The third line assumes IF=LO-RF and is equivalent to the fourth line.

Example:

RF = 325 MHz

LO = 338 MHz

IF = 13 MHz

$\Delta f = 100$ kHz

The RF input of the receiver is sin(325.1 MHz) and the down converted signal at the output is sin(12.9 MHz).

case 2: $\cos(\mathrm{RF} + \Delta f)$
The same derivation can be made in the case where the RF signal is a cosine.

$$\begin{aligned}\cos(\mathrm{RF} + \Delta f) \times \sin \mathrm{LO} &= \frac{1}{2}\sin(\mathrm{RF} + \Delta f + \mathrm{LO}) - \frac{1}{2}\sin(\mathrm{RF} + \Delta f - \mathrm{LO}) \\ &= -\sin(\text{RF-LO} + \Delta f) \\ &= \sin(\mathrm{IF} - \Delta f)\end{aligned}$$

The two cases are summarized in figure 2.

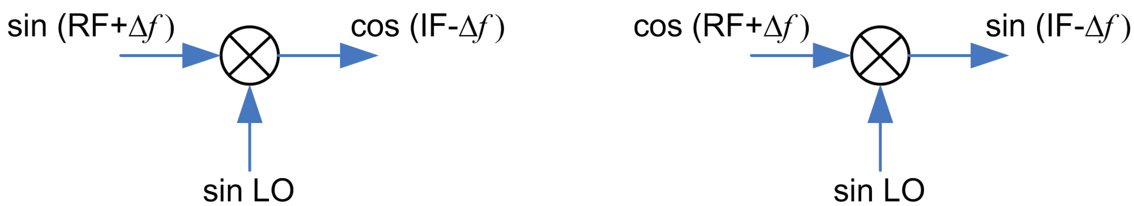

Figure 2: Down converted signal at the output of the receiver

4 Controller down conversion

The digital down conversion implemented in the FPGA consists of splitting the IF input signal and multiplying by cos IF and sin IF to get the I and Q components respectivley, as depicted in figure 3.

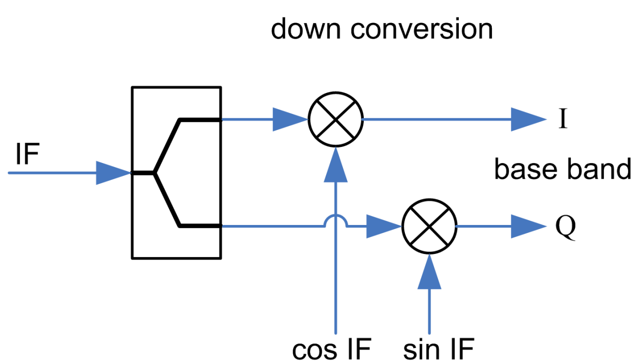

Figure 3: FPGA implementation of the down conversion to I/Q base band components

The expressions for I and Q components are calculated below, assuming a receiver down conversion of $\sin(\mathrm{RF} + \Delta f)$, case 1, and for $\cos(\mathrm{RF} + \Delta f)$, case 2.

case 1: $\sin(\mathrm{RF}+\Delta f) \rightarrow \cos(\mathrm{IF}-\Delta f)$

$$\begin{aligned} I &= \cos(\mathrm{IF}-\Delta f)\times\cos\mathrm{IF} = \frac{1}{2}\cos(\mathrm{IF}-\Delta f+\mathrm{IF}) + \frac{1}{2}\cos(\mathrm{IF}-\Delta f-\mathrm{IF}) \\ &= \cos\Delta f \text{ after filtering} \end{aligned}$$

$$\begin{aligned} Q &= \cos(\mathrm{IF}-\Delta f)\times\sin\mathrm{IF} = \frac{1}{2}\sin(\mathrm{IF}-\Delta f+\mathrm{IF}) - \frac{1}{2}\sin(\mathrm{IF}-\Delta f-\mathrm{IF}) \\ &= \sin\Delta f \text{ after filtering} \end{aligned}$$

Note that the 1/2 coefficient is dropped in the second line for I and Q because this analysis only focuses on the frequency components, not on amplitudes.

case 2: $\cos(\mathrm{RF}+\Delta f) \rightarrow \sin(\mathrm{IF}-\Delta f)$

$$\begin{aligned} I &= \sin(\mathrm{IF}-\Delta f)\times\cos\mathrm{IF} = \frac{1}{2}\sin(\mathrm{IF}-\Delta f+\mathrm{IF}) + \frac{1}{2}\sin(\mathrm{IF}-\Delta f-\mathrm{IF}) \\ &= -\sin\Delta f \text{ after filtering} \end{aligned}$$

$$\begin{aligned} Q &= \sin(\mathrm{IF}-\Delta f)\times\sin\mathrm{IF} = \frac{1}{2}\cos(\mathrm{IF}-\Delta f-\mathrm{IF}) - \frac{1}{2}\cos(\mathrm{IF}-\Delta f+\mathrm{IF}) \\ &= \cos\Delta f \text{ after filtering} \end{aligned}$$

Note that to going from case 1 to case 2 is equivalent to swapping I and Q and inverting one channel. The two cases are summarized in the figure below:

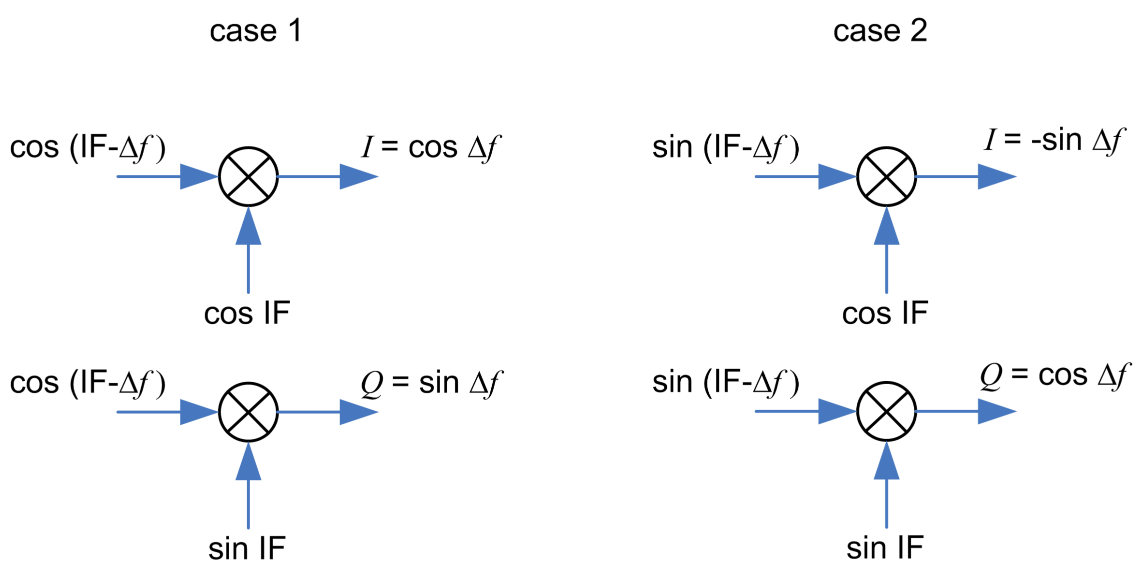

Figure 4: Down conversion in the FPGA illustrated for an RF sine (left) and cosine signal (right)

5 Controller up conversion

This is the up conversion 1 in figure 1, which takes the base band I and Q signal and up converts them to an IF. This operation consists of a complex multiplication, as illustrated in the following figure.

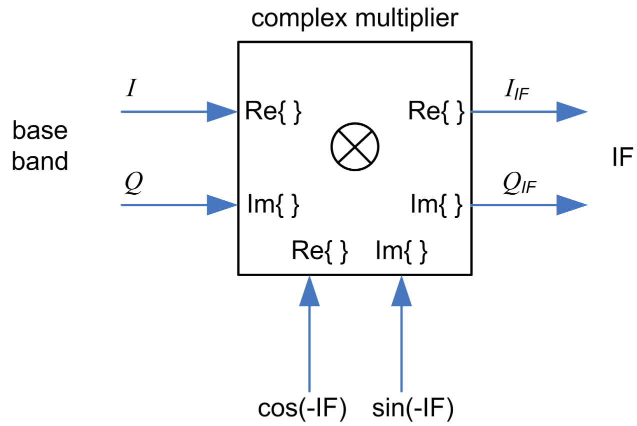

Figure 5: Complex multiplication to perform the up conversion to an IF

The complex multiplication expression is $I_{\mathrm{IF}} + jQ_{\mathrm{IF}} = (I + jQ) \times (\cos(-IF) + j\sin(\text{-IF}))$. The use of a negative frequency -IF for the up conversion is necessary here because the following up conversion to RF (performed by the receiver and described in the following section) makes use of LO=RF+IF. Had we used LO=RF-IF, it would have been necessary to use the positive frequency +IF for the first up conversion performed by the controller.

The mathematical derivation of the complex multiplication is given below:

$$
\begin{aligned}
I_{\mathrm{IF}} &= \mathrm{Re}\{(I + jQ) \times (\cos(\text{-IF}) + j\sin(\text{-IF}))\} \\
&= I\cos(\text{-IF}) - Q\sin(\text{-IF})
\end{aligned}
$$

$$
\begin{aligned}
Q_{\mathrm{IF}} &= \mathrm{Im}\{(I + jQ) \times (\cos(\text{-IF}) + j\sin(\text{-IF}))\} \\
&= I\sin(\text{-IF}) + Q\cos(\text{-IF})
\end{aligned}
$$

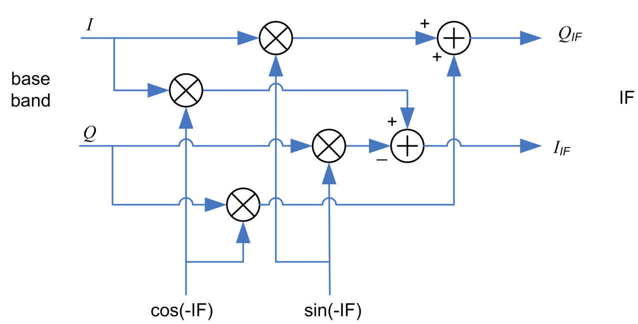

Figure 6: Details of the complex multiplier

case 1: $\sin(\mathrm{RF}+\Delta f) \rightarrow \cos(\mathrm{IF}-\Delta f) \rightarrow \cos\Delta f + j\sin\Delta f$

We consider here the case where an RF signal sin(RF+Δf) was first down converted to IF and then to base band, as described in the previous two sections. In this case, the base band components are I=$\cos\Delta f$ and Q=$\sin\Delta f$. The up conversion operation is described mathematically as:

$$\begin{aligned} I_{\mathrm{IF}} &= \mathrm{Re}\{(I+jQ)\times(\cos(\text{-IF})+j\sin(\text{-IF}))\} \\ &= \cos\Delta f\cos(\text{-IF}) - \sin\Delta f\sin(\text{-IF}) \\ &= \cos(\text{-IF}+\Delta f) \end{aligned}$$

$$\begin{aligned} Q_{\mathrm{IF}} &= \mathrm{Im}\{(I+jQ)\times(\cos(\text{-IF})+j\sin(\text{-IF}))\} \\ &= \cos\Delta f\sin(\text{-IF}) + \sin\Delta f\cos(\text{-IF}) \\ &= \sin(-\mathrm{IF}+\Delta f) \end{aligned}$$

case 2: $\cos(\mathrm{RF}+\Delta f) \rightarrow \sin(\mathrm{IF}-\Delta f) \rightarrow -\sin\Delta f + j\cos\Delta f$

We consider here the case where an RF signal cos(RF+Δf) was first down converted to IF and then to base band, as described in the case 2 of the previous two sections. In this case, the base band components are I=$-\sin\Delta f$ and Q=$\cos\Delta f$. The up conversion operation is described mathematically as:

$$\begin{aligned} I_{\mathrm{IF}} &= \mathrm{Re}\{(I+jQ)\times(\cos(\text{-IF})+j\sin(\text{-IF}))\} \\ &= \sin\Delta f\cos(\text{-IF}) - \cos(\Delta f\sin(\text{-IF}) \\ &= -\sin(-\mathrm{IF}+\Delta f) \end{aligned}$$

$$\begin{aligned} Q_{\mathrm{IF}} &= \mathrm{Im}\{(I+jQ)\times(\cos(\text{-IF})+j\sin(\text{-IF}))\} \\ &= -\sin\Delta f\sin(\text{-IF}) + \cos(\Delta f)\cos(\text{-IF}) \\ &= \cos(-\mathrm{IF}+\Delta f) \end{aligned}$$

Note that the two cases are consistent with each other. The RF signal of case 1 has a 90° phase advance on the RF signal of case 2. Similarly, the base band signal of case 1 has a 90° phase advance on the IF signals of case 2. Finally, the base band signal of case 1 has a 90° phase advance on the base band signal of case 2.

6 Transmitter up conversion

This corresponds to the up conversion 2 on figure 1. The transmitter takes the I and Q signal coming from the controller (at the negative -IF) and up converts these signal to RF. This is implemented with a vector modulator, using an LO signal with LO=RF+IF. The choice of LO (uppper side band versus lower side band) determines whether a positive or a negative IF should be used as an input to the vector modulator. An I and Q signal of +IF used as input to the vector modulator with a LO=RF+IF will result in an output signal of LO+IF=RF+2IF, while a choice of -IF for I and Q will yield an output signal of LO-IF=RF+IF-IF=RF. The later is obviously the correct choice here.

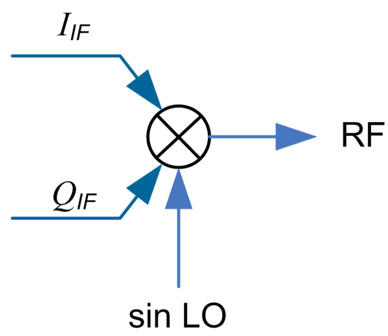

Figure 7: I/Q vector modulator to perform the transmitter up conversion

The mathematical operation modelling the vector modulator consists of multiplying the complex number $I_{\text{IF}} + jQ_{\text{IF}}$ by the LO signal sin LO and filtering the out the undesired side band.

$$RF = (I_{\text{IF}} + jQ_{\text{IF}}) \times \sin \text{LO}$$

case 1: $\sin(\text{RF} + \Delta f) \rightarrow \cos(\text{IF} - \Delta f) \rightarrow \cos \Delta f + j \sin \Delta f \rightarrow \cos(-\text{IF} + \Delta f) + j \sin(-\text{IF} + \Delta f)$

$$\begin{aligned}
RF &= \big(\cos(-\text{IF} + \Delta f) + j \sin(-\text{IF} + \Delta f)\big) \times \sin \text{LO} \\
&= \frac{1}{2}\big(\sin(-\text{IF} + \Delta f + \text{LO}) - \sin(-\text{IF} + \Delta f - \text{LO})\big) + j\frac{1}{2}\big(\cos(-\text{IF} + \Delta f - \text{LO}) - \cos(-\text{IF} + \Delta f + \text{LO})\big) \\
&= \sin(\text{LO-IF} + \Delta f) - j \cos(\text{LO-IF} + \Delta f) \\
&= \sin(\text{RF} + \Delta f) - j \cos(\text{RF} + \Delta f)
\end{aligned}$$

The third line is simplified after filtering out the terms in -LO-IF and dropping the 1/2 amplitude coefficients. The fourth line assumes RF=LO-IF.

case 2: $\cos(\text{RF} + \Delta f) \rightarrow \sin(\text{IF} - \Delta f) \rightarrow -\sin \Delta f + j \cos \Delta f \rightarrow -\sin(-\text{IF} + \Delta f) + j \cos(-\text{IF} + \Delta f)$

$$\begin{aligned}
RF &= \big(-\sin(-\text{IF} + \Delta f) + j \cos(-\text{IF} + \Delta f)\big) \times \sin \text{LO} \\
&= -\frac{1}{2}\big(\cos(-\text{IF} + \Delta f - \text{LO}) + \cos(-\text{IF} + \Delta f + \text{LO})\big) + j\frac{1}{2}\big(\sin(-\text{IF} + \Delta f + \text{LO}) - \sin(-\text{IF} + \Delta f - \text{LO})\big) \\
&= \cos(\text{LO-IF} + \Delta f) + j \sin(\text{LO-IF} + \Delta f) \\
&= \cos(\text{RF} + \Delta f) + j \sin(\text{RF} + \Delta f)
\end{aligned}$$

The third line is simplified after filtering out the terms in -LO-IF and dropping the 1/2 amplitude coefficients. The fourth line assumes RF=LO-IF.

The two cases are summarized in figure 8.

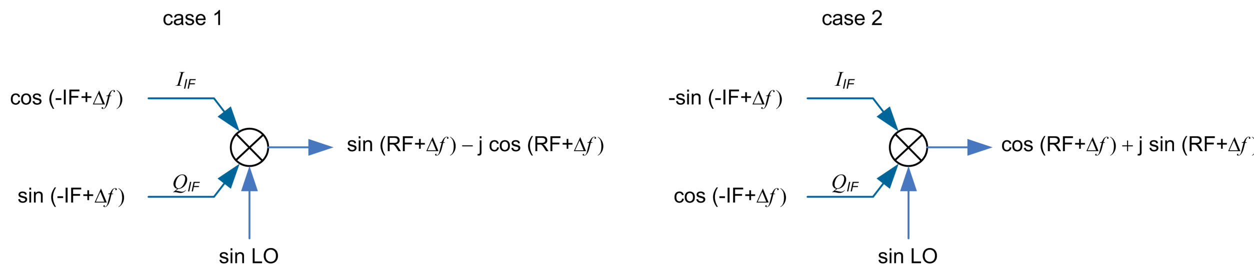

Figure 8: Up converted signal at the transmitter